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Efficient siesmic analysis of building structures with added viscoelastic dampers

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Abstract

Conventional analysis methods for building structures with added viscoelastic dampers, such as direct integration, complex mode superposition, and modal strain energy method, were compared, and a procedure based on rigid diaphragm assumption and matrix condensation technique was proposed for application in the preliminary analysis and design stages. The results from the various analysis methods with and without the matrix condensation were compared, in view of both accuracy and efficiency. According to the eigenvalue analysis the major vibration modes were mostly preserved after the matrix condensation. It was also found that the matrix condensation technique applied to dynamic analysis of a structure with added viscoelastic dampers provided quite accurate results in significantly reduced time, regardless of the plan shape and the location of the viscoelastic dampers.

Keywords: Viscoelastic dampers; Non-proportional damping; Rigid diaphragm; Matrix condensation technique

1. Introduction

For years viscoelastic dampers have been widely used not only to improve residential comfort in strong winds but also to enhance structural safety against large earthquake ground motion. There are many examples, such as World Trade Center in New York and Columbia Center in Seattle, USA, where viscoelastic dampers were applied successfully to enhance the structural performance against dynamic loads [1,2].

Generally, in practice, the behavior of viscoelastic dampers is represented by a spring and a dashpot connected in parallel [3,4]. Although more rigorous methods of analytical modeling exist taking into account the nonlinear behavior of the viscoelastic material, such as based on Boltzmann's superposition principle [5] or on fractional derivative constitutive relationship [6], they may not be applicable for the analysis of large scale structures because of their huge computational demands. With this spring-damper idealization, the dynamic system matrices can be constructed by superimposing the contribution from the dampers onto the system matrices of the structure.

In the dynamic analysis of a building structure subjected to a horizontal earthquake excitation, the damping matrix is generally constructed from a linear combination of the mass and stiffness matrices so that the dynamic equation of motion can be transformed into a set of independent modal equations using the real-valued eigenvectors and eigenvalues of the undamped system. The assumption of proportional damping, however, may not be valid when the viscoelastic dampers are added to the structure. In this case the damping matrix may no longer be proportional to the mass or stiffness matrix, due to the installation of the discrete damping devices. Such a non-proportionally damped structure, with spring-dashpot type added dampers, can accurately be analyzed by the direct integration method or the complex mode superposition method. However the applicability of the methods are limited by inherent shortcomings; the former requires too much computation time and memory space to be applied in practice, and the latter is theoretically complicated because the analysis should be carried out in a complex domain. Also as the number of degrees of freedom (DOFs) increases, the computational time

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required for the eigenvalue analysis increases significantly.

As an alternative the modal strain energy method has been applied for the analysis of a structure installed with viscoelastic dampers [7]. The modal strain energy method derives the equivalent damping ratios based on the assumption of a proportional damping system. However, in the non-proportionally damped system, the responses obtained using the modal strain energy method are essentially approximate, and need to be verified. This approach may not produce accurate results, especially when the added damping is highly localized along the building height.

In this study an efficient analytical procedure is developed for the seismic analysis of structures with added viscoelastic dampers, which are modeled by the linear spring-dashpot connected in parallel. Special attention has been paid to the formation of the system stiffness and damping matrices contributed both from the structure and the added dampers. The results computed from the proposed method are compared with those from the modal strain energy method, direct integration without matrix condensation, and the complex mode superposition method, to check the accuracy and efficiency of the proposed method.

2. Analysis methods for structures with viscoelastic dampers

2.1. Modal strain energy method

The modal strain energy method is a procedure to determine a set of real-valued mode shapes, natural frequencies and damping ratios for a linear structure with frequency-dependent stiffness and damping. In this approach the mode shapes and natural frequencies of the approximate system are obtained by solving an eigenvalue problem that neglects the damping of the structure. Then the modal damping ratios can be estimated by the following equation [7]:

$$\xi_i = \frac{\eta_i}{2}\left[1-\left(\frac{\boldsymbol{\phi}_\mathbf{i}^\mathrm{T}\mathbf{K}_\mathbf{e}\boldsymbol{\phi}_\mathbf{i}}{\boldsymbol{\phi}_\mathbf{i}^\mathrm{T}\mathbf{K}_\mathbf{s}\boldsymbol{\phi}_\mathbf{i}}\right)\right] \tag{1}$$

where ξ_i=equivalent damping ratio for the ith vibration mode, η_i=loss factor of the viscoelastic material, $\mathbf{K}_\mathbf{e}$ =stiffness matrix of the structure without added dampers, $\mathbf{K}_\mathbf{s}$=stiffness matrix of the structure with dampers, and $\boldsymbol{\phi}_\mathbf{i}$=$i$th vibration mode shape of the viscoelastically damped structure. If the change of vibration mode shapes due to the addition of dampers can be neglected, the above equation can be further reduced to:

$$\xi_i = \frac{\eta_i}{2}\left[1-\left(\frac{\omega_i^2}{\omega_{si}^2}\right)\right] \tag{2}$$

where ω_i and ω_{si} are the ith natural frequencies of the structure without and with added dampers, respectively.

The modal strain energy method can be a valuable tool for analysis of structures with a moderate amount of evenly distributed viscoelastic dampers, as shown by Shen et al. [5]. However the method may not be suitable for a structure with a large damping system or unevenly distributed viscoelastic dampers, as will be shown later in the study.

2.2. Complex mode superposition method

The analysis of a structure with spring-damper type viscoelastic dampers can be carried out using the complex mode superposition method, which provides the exact solution for the nonproportionally damped structure [8,9]. As the complex-valued mode vectors satisfy the orthogonality condition, the equation of motion can be uncoupled to modal equations. This method provides accurate solutions regardless of the location of the viscoelastic dampers. Also compared with the direct integration method, the complex mode superposition has the advantage in that the modal characteristics of the non-proportionally damped structure can be identified. However, as the scale of the structure increases the computation time increases rapidly, mainly because the size of the dynamic matrices in eigenvalue analysis is increased to $2n\times 2n$ for n-degrees-of-freedom system. Therefore, a lot of computational time and computer memory is required in the stage of eigenvalue analysis. Furthermore, the procedure is not preferred in practice because it needs complicated numerical procedure in a complex domain.

2.3. Direct integration method

The direct integration method, which computes the responses of a structure by integrating the equation of motion, is commonly used in dynamic analysis of both proportional and non-proportional damping systems. If the direct integration method is based on reasonable numerical operations, precise responses would be obtained regardless of the location of the dampers. However, for structures with a large number of degrees of freedom, a lot of computational time is required because of the iterative nature of the numerical procedure.

3. Development of an efficient analysis method

3.1. Simplified modeling of multistorey structures

Generally floor slabs in a building have very large in-plane stiffness, and the assumption of a rigid diaphragm greatly increases the efficiency of analysis without significant loss of accuracy in estimation of the responses resulting from ground excitations. The rigid floor diaphragm assumption may be most effective for seismic analysis of multistorey buildings.

In addition to the rigid floor diaphragm assumption, the efficiency of computation can be further increased by applying the matrix condensation technique. In this study the in-plane DOFs of all the nodal points located on the floors are concentrated to the three DOFs, representing two translational and one rotational degree of freedom, as described in Fig. 1 (a) and (b). In this way the 3-dimensional model structure is reduced to the stick model, as shown in Fig. 1 (c), which has only three degrees of freedom per floor. In Fig. 1 (a), the original framed structure, the mass of the columns, beams, and slabs are lumped to each nodal point, and the same mass is used, both for the lateral and the rotational degrees-of-freedom. For the structure with floor rigid diaphragms, shown in Fig. 1 (b), the mass moment inertia about the vertical axis passing through the center is utilized for rotational mass.

3.2. Condensation of mass and stiffness matrices

To condense the equation of motion, the degrees of freedom are divided into two parts; the primary ones describing the x and y displacements and the z rotation that will be retained (denoted by the subscript F), and the secondary ones representing the remaining degrees of freedoms to be reduced (denoted by A). Corresponding to the division of the degrees of freedom, the system mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ are divided into four parts, and the equations of motion are expressed as follows:

$$\begin{bmatrix} \mathbf{M_{AA}} & \mathbf{M_{AF}} \\ \mathbf{M_{FA}} & \mathbf{M_{FF}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_\mathbf{A} \\ \ddot{\mathbf{U}}_\mathbf{F} \end{bmatrix} + \begin{bmatrix} \mathbf{K_{AA}} & \mathbf{K_{AF}} \\ \mathbf{K_{FA}} & \mathbf{K_{FF}} \end{bmatrix} \begin{bmatrix} \mathbf{U_A} \\ \mathbf{U_F} \end{bmatrix} = \begin{bmatrix} \mathbf{A_A} \\ \mathbf{A_F} \end{bmatrix} \tag{3}$$

where $\mathbf{U}$ and $\ddot{\mathbf{U}}$ are the displacement and acceleration vectors, respectively, and $\mathbf{A}$ represents the load vector.

According to the Guyan's matrix reduction technique [10], the condensed stiffness and the mass matrices can be written as follows:

$$\mathbf{K}_\mathbf{S}^* = \mathbf{K_{FF}} - \mathbf{K_{FA}}\mathbf{K_{AA}^{-1}}\mathbf{K_{AF}} \tag{4}$$

$$\mathbf{M}_\mathbf{FF}^* = \mathbf{M_{FF}} + \mathbf{T_{AF}^{T}}\mathbf{M_{AF}} + \mathbf{M_{FA}}\mathbf{T_{AF}} + \mathbf{T_{AF}}\mathbf{M_{AA}}\mathbf{T_{AF}} \tag{5}$$

where $\mathbf{T_{AF}}$ is

$$\mathbf{T_{AF}} = -\mathbf{K_{AA}^{-1}}\mathbf{K_{AF}} \tag{6}$$

The load vector is condensed to the following form:

$$\mathbf{A}_\mathbf{F}^* = \mathbf{A_F} - \mathbf{K_{FA}}\mathbf{K_{AA}^{-1}}\mathbf{A_A}. \tag{7}$$

3.3. Formation of the condensed system damping matrix

Construction of the damping matrix for a multi-degree-of-freedom system is somewhat arbitrary, since damping in a building structure is contributed from diverse sources and may not be evaluated precisely. Most commonly recognized methods are Rayleigh damping, constant damping, and modal damping [10]. The Rayleigh damping assumes that the damping is proportional to stiffness and/or mass. The constant damping treats all modal damping ratios as equal, and the modal damping presumes each modal damping ratio.

In the case of Rayleigh damping, the damping matrix is constructed from a linear combination of the mass and the stiffness matrices. Once the damping matrix is ready, it can be condensed the same way as the mass matrix was condensed (Eq. (5)). In this study, however, the condensed structural damping matrix $\mathbf{C}_\mathbf{S}^*$ was obtained by the linear combination of the condensed structural stiffness matrix, $\mathbf{K}_\mathbf{S}^*$, and mass matrix, $\mathbf{M}_\mathbf{S}^*$:

$$\mathbf{C}_\mathbf{S}^* = \alpha\mathbf{M}_\mathbf{S}^* + \beta\mathbf{K}_\mathbf{S}^* \tag{8}$$

where the constant α and β are determined from the two desired modal damping ratios. In this way the computation time and the memory space required in the condensing process can be reduced significantly. This simplified process turned out to be appropriate through the numerical analysis which will be presented later in this study.

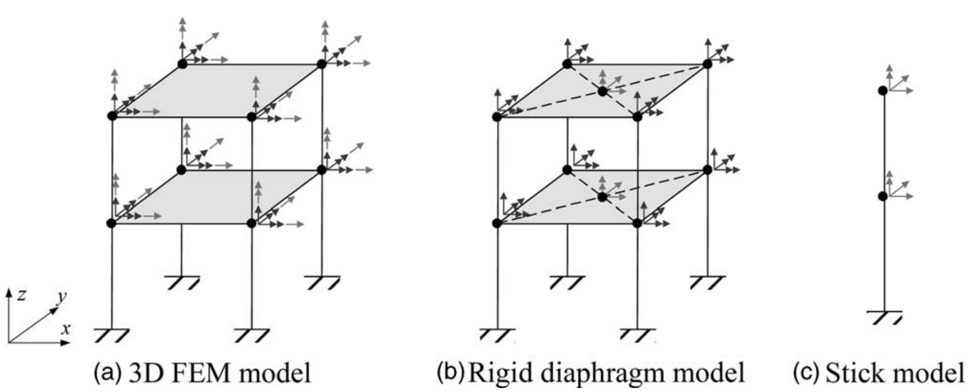


Fig. 1. Description of the rigid diaphragm assumption and the matrix condensation technique.

In the other cases, the modal damping and the constant damping, damping coefficient C_{Ni} in the ith modal equation, which is uncoupled by orthogonal relationship, can be written with the frequency ω_i and the assumed damping ratio ζ_i [10]:

$$C_{Ni} = 2\zeta_i\omega_i \tag{9}$$

Therefore, the condensed damping matrix $\mathbf{C}^*_\mathbf{S}$ is obtained by:

$$\mathbf{C}^*_\mathbf{S} = (\mathbf{\Phi}^{-1})^\mathbf{T}\mathbf{C}_\mathbf{N}\mathbf{\Phi}^{-1} \tag{10}$$

where $\mathbf{\Phi}$ is the mass-normalized mode shape matrix obtained from the eigenvalue analysis with condensed mass and stiffness matrices.

3.4. Condensation of the matrices contributed from the dampers

In the case where the dynamic behavior of viscoelastic dampers is described by an elastic spring and a linear dashpot connected in parallel, as shown in Fig. 2 (a), and when the damper is placed on x–z plane, two degrees of freedom per node are allocated to represent its behavior in the global coordinates. With this idealization the damping and stiffness properties can be added separately to those of the structure. However, for compatibility, the rigid diaphragm assumption and the matrix condensation technique also have to be applied to the viscoelastic dampers before they are added to the condensed system matrices of the structure. Fig. 2 (b) illustrates that with rigid diaphragm assumption, the lateral degrees of freedom are transferred to the mass center of the structure. Then the matrix condensation is applied with the lateral and rotational DOFs as primary ones to be retained and the vertical DOF as a secondary one to be condensed.

The equation of motion associated with the added viscoelastic dampers can be formed as follows:

$$\begin{bmatrix}\mathbf{M}_{\mathbf{DAA}i} & \mathbf{M}_{\mathbf{DAF}i}\\ \mathbf{M}_{\mathbf{DFA}i} & \mathbf{M}_{\mathbf{DFF}i}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{U}}_{\mathbf{DA}i}\\ \ddot{\mathbf{U}}_{\mathbf{DF}i}\end{bmatrix} + \begin{bmatrix}\mathbf{C}_{\mathbf{DAA}i} & \mathbf{C}_{\mathbf{DAF}i}\\ \mathbf{C}_{\mathbf{DFA}i} & \mathbf{C}_{\mathbf{DFF}i}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{U}}_{\mathbf{DA}i}\\ \dot{\mathbf{U}}_{\mathbf{DF}i}\end{bmatrix} + \begin{bmatrix}\mathbf{K}_{\mathbf{DAA}i} & \mathbf{K}_{\mathbf{DAF}i}\\ \mathbf{K}_{\mathbf{DFA}i} & \mathbf{K}_{\mathbf{DFF}i}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{\mathbf{DA}i}\\ \mathbf{U}_{\mathbf{DF}i}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{\mathbf{DA}i}\\ \mathbf{A}_{\mathbf{DF}i}\end{bmatrix} \tag{11}$$

where the quantities with the subscript A are related to the secondary DOFs to be condensed, while those with the subscript B to the primary DOFs to be retained. The subscript D is used to represent that the quantities are associated with the added dampers. To derive the condensed damping matrix for the ith damper, it is assumed that the displacement vectors representing the secondary and primary DOFs are related as follows:

$$\mathbf{U}_{\mathbf{DA}i} = \mathbf{T}_{\mathbf{DAF}i}\mathbf{U}_{\mathbf{DF}i} \tag{12}$$

where $\mathbf{T}_{\mathbf{DAF}i} = -\mathbf{K}^{-1}_{\mathbf{DAA}i}\mathbf{K}_{\mathbf{DAF}i}$ (Eq. (6)). The validity of the assumption can be confirmed by referring to the general procedure of the Guyan's reduction [10]. The derivative of Eq. (12) with respect to time leads to

$$\dot{\mathbf{U}}_{\mathbf{DA}i} = \mathbf{T}_{\mathbf{DAF}i}\dot{\mathbf{U}}_{\mathbf{DF}i} \tag{13a}$$

$$\ddot{\mathbf{U}}_{\mathbf{DA}i} = \mathbf{T}_{\mathbf{DAF}i}\ddot{\mathbf{U}}_{\mathbf{DF}i} \tag{13b}$$

Substituting Eqs. (12) and (13) into Eq. (11) leads to the following condensed equation of motion:

$$\mathbf{M}^*_{\mathbf{D}i}\ddot{\mathbf{U}}_{\mathbf{DF}i} + \mathbf{C}^*_{\mathbf{D}i}\dot{\mathbf{U}}_{\mathbf{DF}i} + \mathbf{K}^*_{\mathbf{D}i}\mathbf{U}_{\mathbf{DF}i} = \mathbf{A}^*_{\mathbf{DF}i} \tag{14}$$

where,

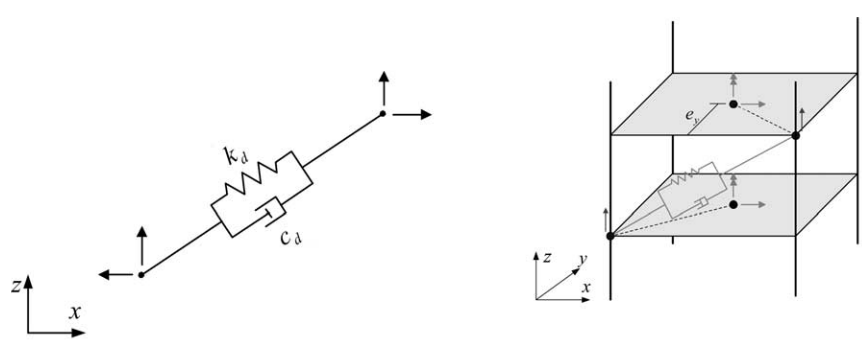
$$\mathbf{K}^*_{\mathbf{D}i} = \mathbf{K}_{\mathbf{DFF}i} - \mathbf{K}_{\mathbf{DFA}i}\mathbf{K}^{-1}_{\mathbf{DAA}i}\mathbf{K}_{\mathbf{DAF}i} \tag{15}$$

$$\mathbf{C}^*_{\mathbf{D}i} = \mathbf{C}_{\mathbf{DFF}i} + \mathbf{T}^\mathbf{T}_{\mathbf{DAF}i}\mathbf{C}_{\mathbf{DAF}i} + \mathbf{C}_{\mathbf{DFA}i}\mathbf{T}_{\mathbf{DAF}i} + \mathbf{T}_{\mathbf{DAF}i}\mathbf{C}_{\mathbf{DAA}i}\mathbf{T}_{\mathbf{DAF}i} \tag{16}$$

$$\mathbf{M}^*_{\mathbf{D}i} = \mathbf{M}_{\mathbf{DFF}i} + \mathbf{T}^\mathbf{T}_{\mathbf{DAF}i}\mathbf{M}_{\mathbf{DAF}i} + \mathbf{M}_{\mathbf{DFA}i}\mathbf{T}_{\mathbf{DAF}i} + \mathbf{T}_{\mathbf{DAF}i}\mathbf{M}_{\mathbf{DAA}i}\mathbf{T}_{\mathbf{DAF}i} \tag{17}$$

$$\mathbf{A}^*_{\mathbf{DF}i} = \mathbf{A}_{\mathbf{DF}i} - \mathbf{K}_{\mathbf{DFA}i}\mathbf{K}^{-1}_{\mathbf{DAA}i}\mathbf{A}_{\mathbf{DA}i} \tag{18}$$

The condensed mass matrix $\mathbf{M}^*_{\mathbf{D}i}$ contributed from the



(a) Spring-dashpot model for a viscoelastic damper in the global x-z coordinates
(b) Transfer of degrees of freedom for rigid diaphragm assumption

Fig. 2. Rigid diaphragm assumption and matrix condensation technique applied for a viscoelastic damper.

dampers can be neglected because the mass of the dampers is very small compared to the structural mass. Comparison of Eqs. (15) and (16) with Eqs. (4) and (5) confirms that the condensed stiffness and damping matrices of the added dampers can be obtained in the same way as the structure stiffness and mass matrices are condensed, respectively.

3.5. Assembly of system matrices

In the previous section, the viscoelastic dampers were condensed separately from the structure. Then the condensed mass, stiffness, and damping matrices of the combined system can be obtained by assembling the condensed matrices of the two different parts. Fig. 3 shows the procedure that combines the condensed stiffness matrix $\mathbf{K}^*_{\mathbf{D}i}$ and damping matrix $\mathbf{C}^*_{\mathbf{D}i}$ of the ith viscoelastic damper to the condensed structural stiffness matrix $\mathbf{K}^*_{\mathbf{S}}$ and the damping matrix $\mathbf{C}^*_{\mathbf{S}}$, respectively. The $\mathbf{K}^*_{\mathbf{FF}}$ and $\mathbf{C}^*_{\mathbf{FF}}$ denote the condensed stiffness and damping matrices of the combined system. In Eqs. (8) and (10), the condensed damping matrix of the structure, $\mathbf{C}^*_{\mathbf{S}}$, is generally a proportional damping matrix. However the condensed damping matrix of the combined system, $\mathbf{C}^*_{\mathbf{FF}}$, becomes non-proportional, due to the contribution from the viscoelastic dampers, and therefore the mode superposition using the real-valued mode vectors cannot be applied.

As mentioned previously, the mass of the added viscoelastic dampers is neglected and the condensed mass of the structure $\mathbf{M}^*_{\mathbf{S}}$ is regarded as the mass of the combined system:

$$\mathbf{M}^*_{\mathbf{FF}} = \mathbf{M}^*_{\mathbf{S}} \tag{19}$$

Finally the equation of motion of the combined system can be written as follows:

$$\mathbf{M}^*_{\mathbf{FF}}\ddot{\mathbf{U}}_{\mathbf{F}} + \mathbf{C}^*_{\mathbf{FF}}\dot{\mathbf{U}}_{\mathbf{F}} + \mathbf{K}^*_{\mathbf{FF}}\mathbf{U}_{\mathbf{F}} = \mathbf{A}^*_{\mathbf{F}} \tag{20}$$

The above equations of motion can be solved by the direct integration method or mode superposition using the complex mode vectors obtained from the eigenvalue analysis, including the non-proportional damping matrix. For the complex mode superposition method, the following dummy equations are generally introduced to make the number of equations of motion compatible with the number of eigenvalues and eigenvectors, which is $2n$ for an n DOF system:

$$\mathbf{M}^*_{\mathbf{FF}}\dot{\mathbf{U}}_{\mathbf{F}} - \mathbf{M}^*_{\mathbf{FF}}\dot{\mathbf{U}}_{\mathbf{F}} = \mathbf{0} \tag{21}$$

The eigenvalues and the eigenvectors of the combined non-proportional damping system can be obtained from the equation of motion expressed in the following state-space form:

$$\begin{bmatrix} \mathbf{0} & \mathbf{M}^*_{\mathbf{FF}} \\ \mathbf{M}^*_{\mathbf{FF}} & \mathbf{C}^*_{\mathbf{FF}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_{\mathbf{F}} \\ \dot{\mathbf{U}}_{\mathbf{F}} \end{bmatrix} + \begin{bmatrix} -\mathbf{M}^*_{\mathbf{FF}} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}^*_{\mathbf{FF}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}}_{\mathbf{F}} \\ \mathbf{U}_{\mathbf{F}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{A}^*_{\mathbf{F}} \end{bmatrix} \tag{22}$$

4. Application of the proposed method

4.1. Model structures for analysis

To verify the efficiency and accuracy of the proposed method, four types of structures shown in Fig. 4 were analyzed. The Model-A, a 3×1 bay and 10-storey framed structure with a rectangular floor plan, were designed to represent a regular structure. In this case the same amount of viscoelastic dampers is mounted on every floor. The Model-B is the same structure with the dampers placed only on the third, fourth, seventh, and eighth inter storey. It was used for confirming that accuracy of the proposed method was assured, regardless of the location of the dampers. The locations of the devices were chosen by sequentially installing a viscoelastic damper at the floor of maximum storey drifts. The Model-C with irregular floor plan, which may have strong participation of the torsional modes in the dynamic motion, was used to check the accuracy of the matrix condensation technique dealing with the localized placement of dampers in an irregular structure. The Model-D, a framed structure with 20-storeys and 3×3 bays, was prepared, to investigate the efficiency of the proposed method. In normal analysis procedures the analysis time required for the structure will be large compared to those for the other model structures. However, the condensed stick model has only twice as many DOFs as those for the 10-storey structures.

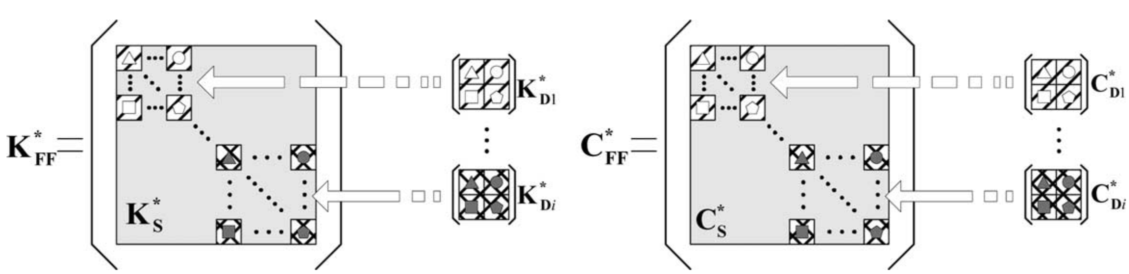


Fig. 3. Assembly of condensed matrices.

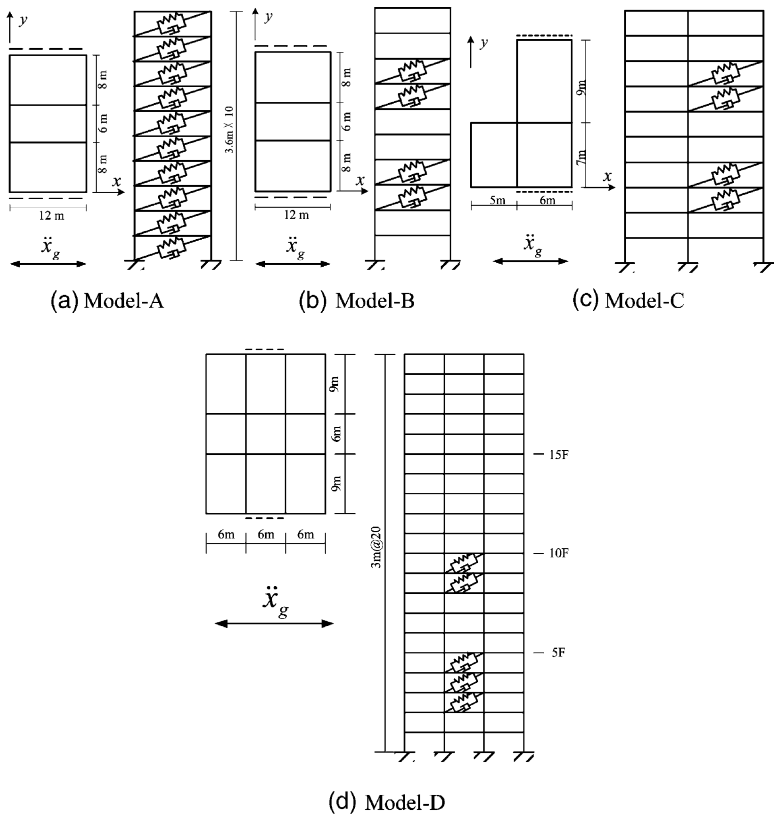


Fig. 4. Model structures for analysis.

Table 1
Lists of the cases for modeling technique and analysis method

Cases		Case-OI	Case-OM	Case-RI	Case-RM	Case-MSE
Matrix condensation		×	×	○	○	×
Analysis method	Direct integration	○	×	○	×	○
	Complex mode superposition	×	○	×	○	×

Table 2(a)
Change of modal characteristics due to matrix condensation. (a) Model-A

Mode	Case-OM		Case-RM		Case-MSE	
	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)
1	0.02	1.06	0.02	1.06	0.21	1.07
2	0.65	1.26	0.62	1.18	0.32	1.45
3	0.74	1.30	0.69	1.22	0.33	1.65
4	0.04	3.24	0.04	3.24	0.15	3.29
5	0.06	5.61	0.06	5.62	0.31	4.58
6	0.09	8.25	0.09	8.25	0.33	5.10
7	0.64	8.60	0.13	11.14	0.04	5.74
8	0.12	11.15	0.16	14.25	0.29	8.16
9	0.66	9.57	0.20	17.45	0.30	8.47
10	0.16	14.26	0.23	20.50	0.05	8.84

Table 2(b)
Model-B

Mode	Case-OM Damping ratio	Frequency (Hz)	Case-RM Damping ratio	Frequency (Hz)	Case-MSE Damping ratio	Frequency (Hz)
1	0.02	1.06	0.02	1.06	0.20	1.06
2	0.27	1.26	0.29	1.23	0.17	1.16
3	0.24	1.43	0.25	1.42	0.17	1.28
4	0.04	3.24	0.04	3.24	0.15	3.29
5	0.18	4.76	0.21	4.88	0.15	3.60
6	0.15	4.98	0.17	5.06	0.17	3.94
7	0.06	5.62	0.06	5.62	0.04	5.74
8	0.15	6.72	0.17	6.84	0.03	5.87
9	0.13	7.09	0.14	7.14	0.08	6.23
10	0.09	8.25	0.09	8.25	0.00	8.47

Table 2(c)
Model-C

Mode	Case-OM Damping ratio	Frequency (Hz)	Case-RM Damping ratio	Frequency (Hz)	Case-MSE Damping ratio	Frequency (Hz)
1	0.02	1.07	0.02	1.07	0.00	1.08
2	0.19	1.63	0.22	1.57	0.16	1.47
3	0.18	1.69	0.20	1.63	0.13	1.60
4	0.03	3.30	0.03	3.30	0.00	3.35
5	0.13	5.70	0.05	5.81	0.15	4.62
6	0.05	5.81	0.17	5.89	0.11	4.94
7	0.16	6.08	0.23	6.24	0.00	5.92
8	0.08	8.63	0.08	8.64	0.06	7.72
9	0.12	8.79	0.14	8.94	0.04	8.42
10	0.15	9.55	1.16	9.72	0.00	8.85

Table 2(d)
Model-D

Mode	Case-OM Damping ratio	Frequency (Hz)	Case-RM Damping ratio	Frequency (Hz)
1	0.02	0.66	0.02	0.66
2	0.06	0.85	0.06	0.84
3	0.07	0.86	0.08	0.86
4	0.03	1.99	0.03	1.99
5	0.10	2.58	0.10	2.57
6	0.10	2.60	0.11	2.59
7	0.05	3.40	0.05	3.40
8	0.16	4.74	0.20	4.68
9	0.16	4.75	0.20	4.76
10	0.07	4.87	0.07	4.87

The columns and girders of all model structures are composed of H-400×400×13×12 and H-300×300×10×13 (unit: mm), respectively. The masses of structural members were lumped to each node, and then were concentrated to the mass center on each floor by rigid diaphragm assumption. Rayleigh damping was utilized to represent the damping of the structure so that the first and the second modal damping ratios be 2%. The material properties of viscoelastic material used in this study were taken from Zhang and Soong [1]; the estimated stiffness and equivalent damping coefficients are k_d=19.0 tonf/cm, c_d=4.0 tonf sec/cm. The 1940 El Centro earthquake NS component was used as an input ground excitation.

Each model structure was analyzed in five different ways differing in modeling technique and analysis method as shown Table 1. The letter ‘I’ represents direct integration method, and ‘M’ means the complex mode superposition method. The letter ‘O’ denotes the case without the matrix condensation, and the letter ‘R’ represents the case with condensation. Finally Case-MSE represents the analysis using the modal strain energy method. The matrix condensation technique was not applied for this case.

4.2. Comparison of modal characteristics

The natural frequencies and modal damping ratios for the model structures with and without matrix condensation are presented in Table 2. Since the direct integration method does not need eigenvalue analysis, only the Case-OM, Case-RM, and Case-MSE were compared. The modal strain energy method was not applied to the model D.

It can be observed that the number of natural modes, which represent the vibration characteristics of a structure, is reduced as a result of the matrix condensation. However, most of the natural frequencies and damping ratios of the major vibration modes in the condensed model, Case-RM, remains quite similar to those of Case-OM in all cases, regardless of the plan shape and the installation of the dampers. Based on these observations, it can be concluded that even if the matrix condensation technique is applied, the principle modes that dominate the dynamic responses are mostly preserved. However the modal characteristics predicted by the modal strain energy method turned out to be very different from those of other cases. This is due to the fact that the modal strain energy method failed to take into account accurately the change in modal characteristics resulting from the addition of the large damping.

4.3. Comparison of time history analysis results

The displacement time histories at the top floor of each model structure are compared in Fig. 5. It can be seen that the results for Case-RM, mode superposition method on a condensed stick model, are very similar to those of Case-OM, mode superposition on original model. This can be expected, considering the similarity in modal characteristics of the condensed and the original models. Likewise the responses for Case-OI and Case-RI, result from the direct integration method with and without condensation, are almost identical. The accuracy of the results from the condensed model is preserved also for structures with arbitrarily distributed viscoelastic dampers and with an irregular plan shape. However, it can be observed that the responses for the case-MSE are quite different from the others.

Figs. 6 and 7 show the maximum storey displacements and inter-storey drifts of the model structures, respectively. A little discrepancy can be observed between the results for the condensed and the original model structures, but the difference is small enough to be negligible in engineering practice. It can also be noticed that the results from the modal strain energy method highly underestimate those predicted from the other methods.

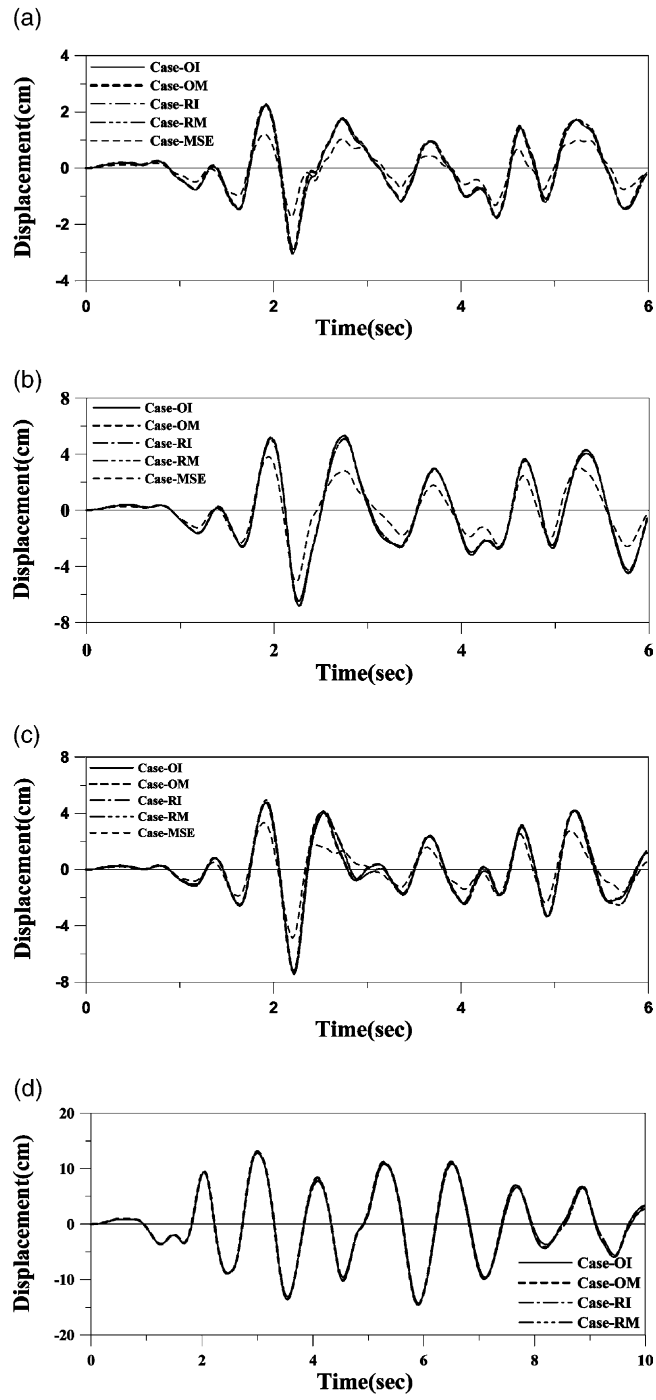


Fig. 5. Roof displacement time histories. (a) Model-A; (b) Model-B; (c) Model-C; (d) Model-D.

In previous examples, the first and the second modal damping ratios were assumed to be 2% of the critical damping, which is generally applied for structures behaving elastically under moderate earthquake or wind

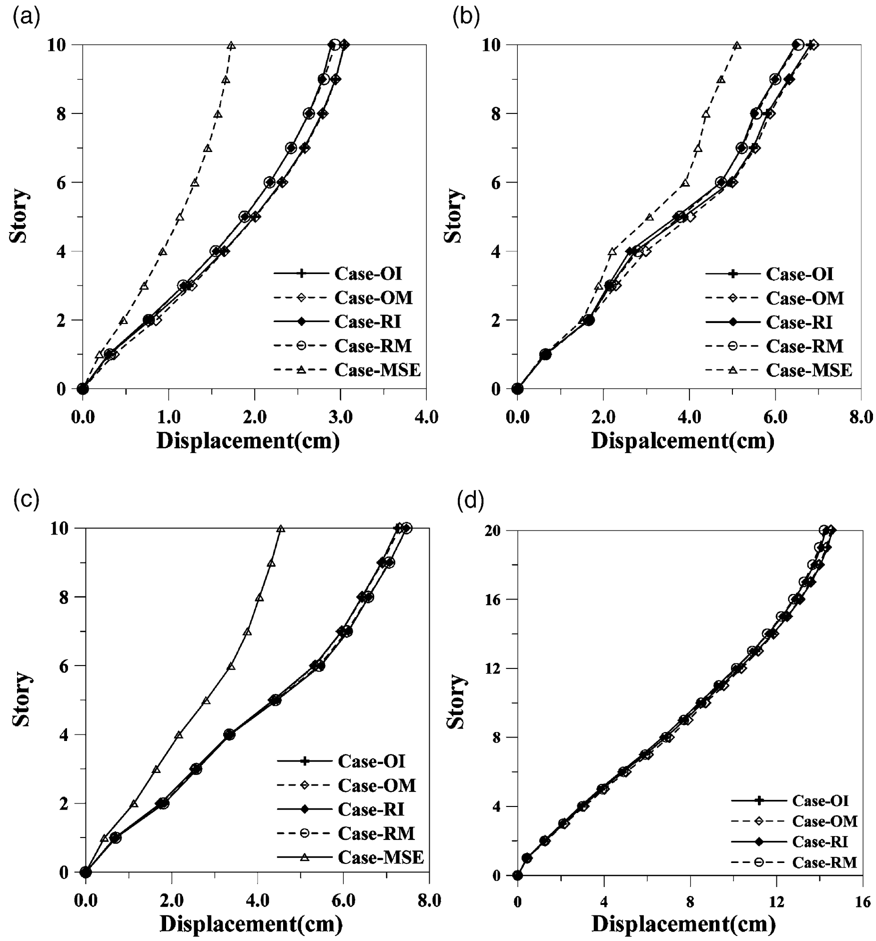


Fig. 6. Maximum storey displacements. (a) Model-A; (b) Model-B; (c) Model-C; (d) Model-D.

load. Fig. 8 presents the analysis results for the Model-B with the modal damping ratios increased to 5%, which is generally used for structures subjected to strong earthquake, and the results were compared with those with 2% damping. It can be noticed that the accuracy of the proposed method is preserved in structures with a higher damping ratio.

4.4. Comparison of computation time

The computation time required for each analysis method is compared in Table 3 for model structure A and D. As the numbers of degrees of freedom of structures A, B and C are the same, only structure A is taken for analysis. In case A, as shown in Table 3 (a), the number of degrees-of-freedom and the matrix size in the condensed stick model were reduced to 1/16 and 1/256, respectively. Consequently even though additional computation time was needed in the matrix condensation procedure in the Case-RI and RM, a lot of analysis time and memory space can be saved in the overall dynamic analysis procedure. The computational time for Case-RI was reduced to only about 0.1% of that required for Case-OI. This resulted from the significant reduction in DOFs by matrix condensation. The Case-RM with reduced DOFs consumed less than 1% of the analysis time compared to Case-OM. The time required for Case-MSE is between the cases with the original and the reduced models.

The efficiency of the matrix condensation technique turned out to be more considerable in the Model-D as can be seen in Table 3 (b). In the Model-D the number of stories was doubled, while the number of DOFs increased four times. However, in case the matrix condensation technique is applied, the number of DOFs of the reduced stick model of the Model-D increases twice because three DOFs exist per floor, regardless of the number of bays. As expected, this reduction in DOFs resulted in significant saving in computation time; the Case-RI and Case-RM required less than 0.05% of the analysis time needed for Case-OI and Case-OM. Furthermore, a lot of computer memory space could be saved because the matrix size was reduced to 1/1024. Throughout the study a personal computer equipped with Pentium-III 500 MHz main board and 128 Mbyte RAM was used in the dynamic analysis.

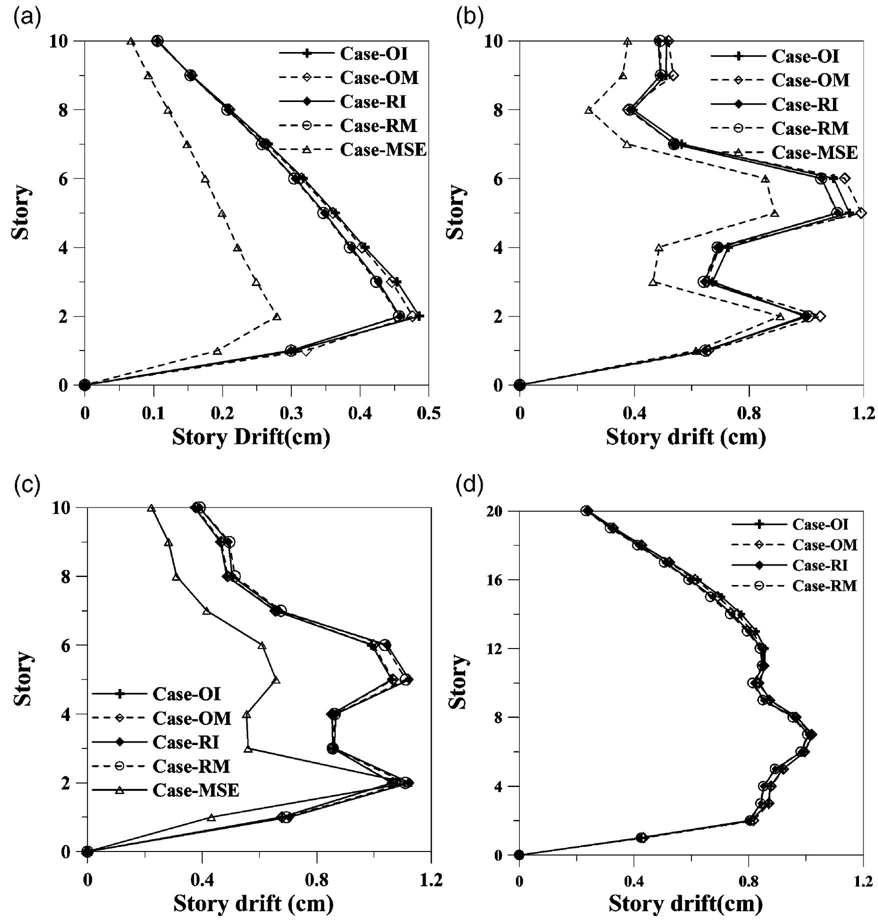


Fig. 7. Maximum inter-storey drifts. (a) Model-A; (b) Model-B; (c) Model-C; (d) Model-D.

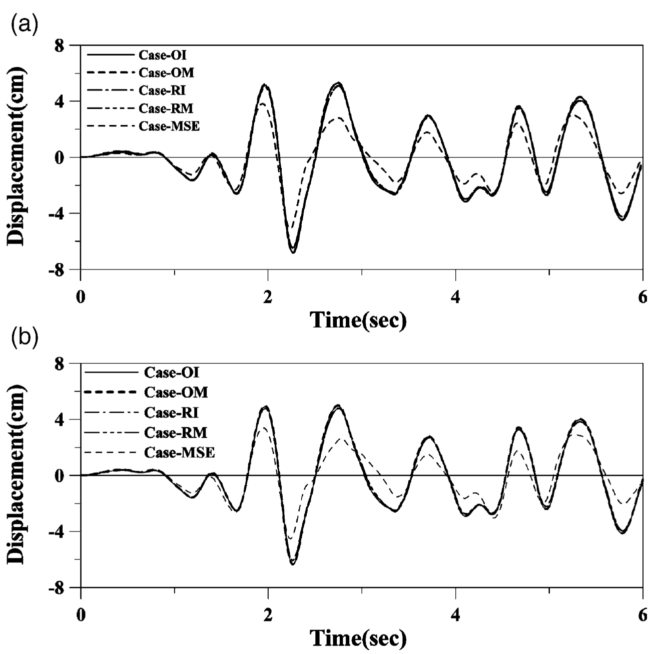


Fig. 8. Roof displacement time histories of Model-B. (a) With 2% inherent damping; (b) With 5% inherent damping.

5. Conclusions

In this study an efficient analytical procedure, based on matrix condensation technique, was proposed for the seismic analysis of structures with added viscoelastic dampers. Special attention has been paid to the condensation of the stiffness and damping matrices contributed from the added dampers. The results from the modal strain energy method, direct integration and the complex mode superposition method without matrix condensation were compared with the results from the proposed method to check the reliability and efficiency of the proposed method.

According to the eigenvalue analysis the major vibration modes were mostly preserved after the matrix condensation. It was also found that the matrix condensation technique applied to dynamic analysis of a structure with added viscoelastic dampers provided accurate results in significantly reduced time, regardless of the location of the viscoelastic dampers. In the direct integration method the largest benefit of matrix condensation could be observed in the process of time history analysis, while in the mode superposition method the reduction in computation time was most prominent in the process

Table 3(a)
Number of DOF's and required computation time (unit: second). (a) Model-A

Cases	Case-OI	Case-OM	Case-RI	Case-RM	Case-MSE
DOF's	480	480	30	30	480
Matrix size	480×480	960×960	30×30	60×60	480×480
Condensation	–	–	2.2	2.3	
Eigenvalue analysis	–	417.1	–	0.3	62.4
Time history analysis	5125.0	497.1	4.7	2.9	291.5
Total	5125.0	914.2	6.9	5.5	353.9

Table 3(b)
Model-D

Cases	Case-OI	Case-OM	Case-RI	Case-RM
DOF's	1920	1920	60	60
Matrix size	1920×1920	3840×3840	60×60	120×120
Condensation	–	–	113.8	114.4
Eigenvalue analysis	–	26694.4	–	2.0
Time history analysis	291240.6	35349.3	37.1	22.9
Total	291240.6	62043.7	150.9	139.3

of eigenvalue analysis. The efficiency of the proposed method is expected to increase as the scale of the structure increases. Finally the direct integration or mode superposition method combined with the matrix condensation technique, turned out to provide more accurate results in less computation time compared with the modal strain energy method. Based on these findings it could be concluded that the proposed procedure can be a useful tool for predicting the dynamic behavior of a structure with viscoelastic dampers, especially in the stage of preliminary analysis and design, or in the process of determining the optimum amount and location of the supplemental dampers.

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